



**UNIVERSITI PUTRA MALAYSIA**

**PARALLEL BLOCK METHODS FOR SOLVING HIGHER ORDER  
ORDINARY DIFFERENTIAL EQUATIONS DIRECTLY**

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**FSAS 1999 4**

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By

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Thesis Submitted in Fulfillment of the Requirement for the  
Degree of Doctor of Philosophy in the Faculty of  
Science and Environmental Studies  
Universiti Putra Malaysia

December 1999



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirements for the degree of Doctor of Philosophy.

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December 1999

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Faculty: Science and Environmental Studies

Numerous problems that are encountered in various branches of science and engineering involve ordinary differential equations (ODEs). Some of these problems require lengthy computation and immediate solutions. With the availability of parallel computers nowadays, the demands can be achieved.

However, most of the existing methods for solving ODEs directly, particularly of higher order, are sequential in nature. These methods approximate numerical solution at one point at a time and therefore do not fully exploit the capability of parallel computers. Hence, the development of parallel algorithms to suit these machines becomes essential.

In this thesis, new explicit and implicit parallel block methods for solving a single equation of ODE directly using constant step size and back values are developed. These methods, which calculate the numerical solution at more than one point simultaneously, are parallel in nature. The programs of the methods employed are run on a shared memory Sequent Symmetry S27 parallel computer. The numerical results show that the new methods reduce the total number of steps and execution time. The accuracy of the parallel block and 1-point methods is comparable particularly when finer step sizes are used.

A new parallel algorithm for solving systems of ODEs using variable step size and order is also developed. The strategies used to design this method are based on both the Direct Integration (DI) and parallel block methods. The results demonstrate the superiority of the new method in terms of the total number of steps and execution times especially with finer tolerances.

In conclusion, the new methods developed can be used as viable alternatives for solving higher order ODEs directly.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi syarat untuk ijazah Doktor Falsafah

**KAEDAH BLOK SELARI BAGI MENYELESAIKAN PERSAMAAN  
PEMBEZAAN BIASA PERINGKAT TINGGI SECARA LANGSUNG**

Oleh

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Pelbagai masalah yang melibatkan persamaan pembezaan biasa ditemui dalam bidang sains dan kejuruteraan. Sesetengah masalah tersebut memerlukan pengiraan yang panjang dan penyelesaian segera. Dengan adanya komputer selari pada masa kini, kedua-dua tuntutan tersebut dapat dipenuhi.

Walau bagaimanapun, kebanyakan kaedah yang sedia wujud bagi menyelesaikan persamaan pembezaan biasa secara langsung, terutamanya yang berperingkat tinggi, adalah bersifat jujukan. Kaedah tersebut mengganggu penyelesaian pada satu titik pada satu masa dan oleh itu tidak memanfaatkan keupayaan komputer selari dengan sepenuhnya. Oleh yang demikian, pembangunan algoritma selari yang sesuai dengan komputer tersebut amatlah diperlukan.

Dalam tesis ini kaedah baru blok selari tersirat dan juga tak tersirat bagi menyelesaikan pembezaan biasa tunggal dengan menggunakan saiz langkah dan nilai belakang malar dibangunkan. Kaedah ini yang menghitung penyelesaian berangka pada beberapa titik serentak adalah bersifat selari. Semua atur cara dilaksanakan dengan menggunakan Sequent Symmetry S27 iaitu sebuah komputer selari berkongsi ingatan. Keputusan berangka menunjukkan kedua-dua kaedah baru ini dapat mengurangkan bilangan langkah dan masa pelaksanaan. Kejutuan kaedah blok selari dan 1-titik adalah boleh banding khususnya bila saiz langkah kecil digunakan.

Satu algoritma baru bagi menyelesaikan sistem persamaan pembezaan dengan menggunakan saiz langkah dan nilai belakang boleh berubah turut diperkenalkan. Strategi yang digunakan bagi merekabentuk kaedah ini adalah berasas kepada kaedah Pengamiran Langsung dan blok selari. Keputusan berangka membuktikan kelebihan kaedah baru ini dari segi bilangan langkah dan masa pelaksanaan terutamanya bagi toleransi yang kecil.

Kesimpulannya, kaedah baru yang dibangunkan boleh diguna sebagai alternatif dalam penyelesaian persamaan pembezaan biasa peringkat tinggi secara langsung.

## ACKNOWLEDGEMENTS

*In the Name of Allah  
The Most Beneficent, The Merciful*

This thesis would not have been possible without the help and support of many people. My sincere and deepest gratitude to Prof. Dr. Mohamed Suleiman, Chairman of the Supervisory Committee, for his outstanding supervision and continuous support. He has been very understanding and always willing to help.

Special thanks are due to the members of the Supervisory Committee, Assoc. Prof. Dr. Bachok Taib and Dr. Yazid Md. Saman, for their assistance and encouragement. I would also like to thank my friends and colleagues for their support and encouragement.

I am indebted to the Government of Malaysia, particularly Universiti Utara Malaysia, for granting the scholarship and study leave to pursue my studies. My gratitude is also extended to my parents, Omar Bakar and Mariam Abdullah, mother in law, Wan Chik Khalid, and family for being so supportive and helpful.

Last but not least, my sincere and special thanks to my wife, Shahida and children, Nur Izni, Nur Hani, Nur Hana, Nur Lina and Ahmad Arif for their understanding, caring, patience and continuous love.

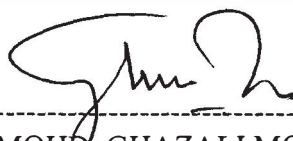
I certify that an Examination Committee has met on 18 December, 1999 to conduct the final examination of Zurni bin Omar, on his Doctor of Philosophy thesis entitled “Parallel Block Methods For Solving Higher Order Ordinary Differential Equations Directly” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommended that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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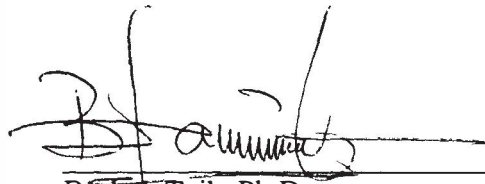
Adalah disahkan bahawa saya telah membaca tesis ini bertajuk “Parallel Block Methods For Solving Higher Order Ordinary Differential Equations Directly” oleh Zurni bin Omar, dan berpendapat bahawa tesis ini adalah memuaskan dari segi skop, kualiti dan persembahan sebagai memenuhi syarat keperluan ijazah Doktor Falsafah.



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Adalah disahkan bahawa saya telah membaca tesis ini bertajuk “Parallel Block Methods For Solving Higher Order Ordinary Differential Equations Directly” oleh Zurni bin Omar, dan berpendapat bahawa tesis ini adalah memuaskan dari segi skop, kualiti dan persembahan sebagai memenuhi syarat keperluan ijazah Doktor Falsafah.



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